Stochastic Versions of Logarithmic Sobolev Inequalities

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Abstract

Our main results refer to the stochastic versions of the well-known Poincare and logarithmic Sobolev inequalities. The Poincare inequality for Gaussian variables was formulated in connection with classical isoperimetric problem. The log-Sobolev inequality was established by Gross in 1975 as an equivalent condition for hypercontractivity of associated Markov semigroup. This work gave start for the further research. Log-Sobolev inequalities are closely linked with transport and entropy-information inequalities etc, they were intensively studied in probability theory, geometry, statistical mechanics [2], [3].

We extend the results from Gaussian case to infinitely divisible variables with indication of optimal constants. Method applied is based on the idea of "embedding" such variables in infinitely divisible process, its dynamics allows us to use Ito's formula and Kolmogorov equations. Estimates are obtained in terms of triplet of local characteristics. The principal advantage of log-Sobolev inequalities is dimension independence (for example, standard Sobolev inequalities are not dimension independent). The method of proof based on stochastic calculus and integral representations works naturally on path spaces [1].

We obtain the Poincare and the log-Sobolev inequality for the skew Brownian motion. The skew Brownian motion is a unique strong solution of a stochastic equation $X_t = B_t + (2\alpha - 1)L_t^0(X), t \geq 0, (\alpha \in [0,1])$, where $L^0(X)$ is the local time of the unknown process X at zero. Such generalized diffusion can be used to model permeable barrier. Drift coefficient is not defined at zero and this effect is translated into the backward equation as a transmission condition, we have to apply generalized change of variable (Ito-Tanaka) formula etc. The estimates depend on the local time of the process. Different points of view on skew Brownian motion, its applications and generalizations can be found in Lejay[4].

Effectiveness of log-Sobolev inequalities in infinite dimensional analysis is illustrated in statistical mechanics and analysis on path spaces. Different versions of log-Sobolev inequalities are appeared to be useful in probability, differential equations, combinatorics. Being a standard application of log-Sobolev type inequalities concentration of measure phenomenon is useful in statistics as well as in geometry.

Keywords: Poincaré inequality, logarithmic Sobolev inequality, skew Brownian motion, Malliavin calculus, Concentration of measure.
AMS subject classifications: 60E15, 60J60, 60H07.

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