Some properties of a class of continuous time moving average processes

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Abstract

In discrete time, moving average processes play an important role in time series analysis. A moving average is a process $\{X_n\}_{n\in\mathbb{N}}$ of the form $X_n = \sum_{k=-\infty}^{n} \phi_{n-k} Z_k$ where $\{\phi_k\}_{k\in\mathbb{N}}$ is a deterministic sequence of real numbers and $\{Z_k\}_{k\in\mathbb{Z}}$ is a sequence of independent and identically distributed random variables. In continuous time, moving averages are processes $X = \{X_t : t \in \mathbb{R}_+\}$ of the form

$$X_t = \int_{-\infty}^{t} \phi(t-s)\, dZ_s \tag{2}$$

where $\phi : \mathbb{R}_+ \to \mathbb{R}$ is a deterministic function and $Z = \{Z_t : t \in \mathbb{R}\}$ is a process with stationary and independent increments (a so-called Lévy process). In this work we will consider a continuous time moving average X of the form (2) in the the case where the kernel function ϕ is the gamma density, i.e. $\phi(t) = e^{-\lambda t} t^{\gamma-1}$. We will derive necessary and sufficient conditions for X to be well-defined, that is, for the existence of the stochastic integrals (2). In some cases X has very irregular sample paths, e.g. they are unbounded on every bounded interval. We give necessary and sufficient conditions for X to have the following type of regularity: almost all sample paths are of bounded variation, or more generally, the process is a semimartingale. These two condition corresponds to that stochastic integrals of the form $\int_0^t Y_s\, dX_s$ are well-defined in the Lebesgue–Stieltjes sense or in the Itô sense, respectively. Our work uses the recent results [2, 3, 4]. Finally let us mention that the gamma kernel has been extensively used to build stochastic models for turbulence; see [1] and the reference therein.

Keywords: Moving averages, gamma density, bounded variation, semimartingales
AMS subject classifications: 60G48; 60H05; 60G51; 60GH17

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