Constructing hierarchical copulas using the Kendall distribution function

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Abstract

While there is substantial need for dependence models in higher dimensions, most existing models are rather restrictive and barely balance parsimony and flexibility. In this talk, the class of hierarchical Kendall copulas is proposed as a new approach to tackle these problems. By aggregating dependence information of non-overlapping groups of variables in different hierarchical levels using the Kendall distribution function, hierarchical Kendall copulas provide a new and attractive option to model dependence patterns between large numbers of variables
In particular, let $U_1, ..., U_n$ be uniform random variables, C_0 a d-dimensional copula and $C_1, ..., C_d$ copulas of dimension $n_1, ..., n_d$, where $n = \sum_{i=1}^{d} n_i$. Further, define $m_i = \sum_{j=1}^{i} n_j$ for $i = 1, ..., d$ and $m_0 = 0$. Then the random vector $(U_1, ..., U_n)'$ is said to be distributed according to the two-level hierarchical Kendall copula $C_{\mathcal{K}}$ with nesting copula C_0 and cluster copulas $C_1, ..., C_d$ if

1. $(U_{m_{i-1}+1}, ..., U_{m_i})' \sim C_i$ for all $i = 1, ..., d$, and
2. $(V_1, ..., V_d)' \sim C_0$, where $V_i := K_i(C_i(U_{m_{i-1}+1}, ..., U_{m_i})) \sim U(0, 1)$ for all $i = 1, ..., d$ and K_i denotes the Kendall distribution function corresponding to C_i, that is the multivariate probability integral transform

$$K_i(t) := P(C_i(U_{m_{i-1}+1}, ..., U_{m_i}) \leq t) \quad \text{for } t \in [0, 1].$$

This mimics the standard copula approach with univariate margins. The cluster copulas $C_1, ..., C_d$ and the nesting copula C_0 can be chosen independently from any class of copulas. Furthermore, the definition can also easily be extended to an arbitrary number of levels.
The talk explicitly discusses properties as well as inference techniques for hierarchical Kendall copulas, in particular, simulation, estimation and model selection. A closed-form sampling algorithm is derived for Archimedean, Plackett and Archimax copulas, while for general copulas an approximative method is proposed. For estimation, a sequential and a joint approach are discussed and compared in an extensive simulation study.
In an application, a three-level hierarchical Kendall copula will be used to conduct a systemic risk stress testing exercise of the international financial sector, providing a ranking of systemically important institutions.

Keywords: multivariate copula, hierarchical copula, Kendall distribution function
AMS subject classifications: 62H20

Bibliography

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