## Estimation of extreme risk regions under multivariate regular variation

Juan-Juan Cai<sup>1</sup> and John H.J. Einmahl<sup>1</sup> and Laurens de Haan<sup>2</sup>

 $$^1{\rm Tilburg}$  University  $^2{\rm Tilburg}$  University, Lisbon University, Erasmus University, Rotterdam

## Abstract

When considering d possibly dependent random variables one is often interested in extreme risk regions, with very small probability p. We consider risk regions of the form  $\{\mathbf{z} \in \mathbb{R}^d : f(\mathbf{z}) \leq \beta\}$ , where f is the joint density and  $\beta$  a small number. Estimation of such an extreme risk region is difficult since it contains hardly any or no data. Using extreme value theory, we construct a natural estimator of an extreme risk region and prove a refined form of consistency, given a random sample of multivariate regularly varying random vectors. In a detailed simulation and comparison study the good performance of the procedure is demonstrated. We also apply our estimator to financial data.

**Keywords:** Extremes, level set, multivariate quantile, rare event, spectral density **AMS subject classifications:** 62G32, 62G05, 62G079.