The zero area Brownian bridge

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Abstract

We consider the Brownian motion W on the interval $[0,1]$. The Brownian bridge B arises from the Brownian motion by pinning W_1 down to 0, i.e., the Brownian bridge arises by conditioning the Brownian motion to fulfill $W_1 = 0$. We condition the Brownian bridge further by requiring $\int_0^1 B_s ds = 0$. We call the resulting Gaussian process on $[0,1]$ zero area Brownian bridge and denote it by M. We will study properties of M and give anticipative as well as non-anticipative representations. Our main tool to access the zero area Brownian bridge is to consider the associated operator $u : L_2([0,1]) \to C([0,1])$ of the Brownian motion, where u is given by

$$(uf)(s) = \int_0^s f(x)dx, \qquad f \in L_2([0,1]).$$

Then (a version of) the Brownian motion is given by

$$W_s = \sum_{i=0}^{\infty} \xi_i (ue_i)(s),$$

where $(e_i)_i$ is an orthonormal basis in $L_2([0,1])$ and $(\xi_i)_i$ is a series of independent standard normal random variables (cf. [2]). The zero area Brownian bridge is the Gaussian process associated to the operator $v : H \to C([0,1])$, where v is the restriction of u to a suitable subspace $H \subset L_2([0,1])$.

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