Hidden Markov models in modeling time series of earthquakes

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Abstract

Discrete valued Hidden Markov Models (HMMs) are used to model time series of event counts in several scientific fields like genetics, engineering, seismology and finance ([1]). Typically the model consists of two parts: the observed sequence of event counts and an unobserved (hidden) sequence of states that consist a Markov chain. In the univariate case each state is characterized by a different univariate distribution and the progress of the Hidden process from state to state is controlled by a transition probability matrix. Conditional on the state of the model at a specific point of time, the distribution of the observation at that point is fully specified. We have assumed both Poisson (PHMMs) and negative binomial (NBHMMs) distribution to dominate each state. An extension of the univariate HMMs to the multivariate case can be made by assuming a multivariate discrete distribution associated with each state. The known as multivariate Poisson distribution in [2], is incorporated into the model to allow as modeling multivariate discrete valued time series. We examine properties of the model and propose inference. Maximum likelihood estimators of the models' parameters are derived using an EM algorithm. Univariate PHMMs and NBHMMs and a bivariate Poisson HMM are applied to earthquake data from Sumatra, Indonesia. On 26 December 2004 and 28 March 2005 occurred two of the largest earthquakes of the last 40 years between the Indo-Australian and the southeastern Eurasian plates with moment magnitudes $Mw = 9.1$ and $Mw = 8.6$ respectively. HMMs models are used for identifying temporal patterns in the time series of the two mainshocks. Each time series consists of earthquake counts, in different time units (days, two-day periods, five-day periods), in the regions determined by the aftershock zones of the two mainshocks. In addition, a bivarite Poisson HMM is applied to jointly model the two time series and the correlation between them is estimated.

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