Drift parameter estimation in models with fractional Brownian motion by discrete observations

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Abstract

Let $B^H = \{B_t^H, t \geq 0\}$ be fractional Brownian motion with Hurst index $H \in (1/2, 1)$. We study the problem of estimating of unknown drift parameter from [1]. Consider the stochastic differential equation driven by fractional Brownian motion B^H:

$$\begin{aligned} dX_t &= \theta a(X_t)dt + b(X_t)dB_t^H, \quad 0 \leq t \leq T, \quad T > 0, \\ X\big|_{t=0} &= X_0 \in \mathbb{R}. \end{aligned} \tag{1}$$

Here $\theta \in \mathbb{R}$ is unknown parameter to be estimated.
Suppose that we observe values $X_{\frac{k}{2^n}}$, $k = 0, 1, \ldots, 2^{2n}$. Consider two estimators for θ:

$$\hat{\theta}_n^{(1)} = \frac{\sum_{k=1}^{2^{2n}} \left(\frac{k}{2^n}\right)^{-\alpha} \left(2^n - \frac{k}{2^n}\right)^{-\alpha} b^{-1}\left(X_{\frac{k-1}{2^n}}\right)\left(X_{\frac{k}{2^n}} - X_{\frac{k-1}{2^n}}\right)}{\sum_{k=1}^{2^{2n}} \left(\frac{k}{2^n}\right)^{-\alpha} \left(2^n - \frac{k}{2^n}\right)^{-\alpha} b^{-1}\left(X_{\frac{k-1}{2^n}}\right) a\left(X_{\frac{k-1}{2^n}}\right)\frac{1}{2^n}},$$

$$\hat{\theta}_n^{(2)} = \frac{\sum_{k=1}^{2^{2n}} b^{-1}\left(X_{\frac{k-1}{2^n}}\right)\left(X_{\frac{k}{2^n}} - X_{\frac{k-1}{2^n}}\right)}{\sum_{k=1}^{2^{2n}} b^{-1}\left(X_{\frac{k-1}{2^n}}\right) a\left(X_{\frac{k-1}{2^n}}\right)\frac{1}{2^n}}.$$

In the simplest cases (for example, $a = b$) the estimator $\hat{\theta}_n^{(1)}$ coincides with a discrete version of maximum-likelihood estimator. $\hat{\theta}_n^{(2)}$ is a non-standard estimator. We prove that both estimators converge to the true value of the parameter θ.

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Bibliography

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