Adaptive Bayesian estimation in Gaussian sequence space models

Jan Johannes[1], **Rudolf Schenk**[1] **and Anna Simoni**[2]

[1] *Université catholique de Louvain, Voie du Roman Pays 20, B-1348 Louvain-la-Neuve*
[2] *Université de Cergy-Pontoise, 33 boulevard du Port, F - 95011 Cergy-Pontoise*

Abstract

We consider the inverse regression problem $Y = Af + \sqrt{\varepsilon}\xi$, where A is a known linear operator between two Hilbert spaces, ξ a Gaussian white noise, and ε a noise level. The objective of this paper is to establish adaptive nonparametric posterior concentration rates of convergence for the regression function f. In a first step, we derive lower and upper bounds for the posterior concentration rates over a family of Gaussian prior distributions indexed by a tuning parameter. These rates are based on tail bounds for noncentral χ^2 distributions established in [1]. By selecting the optimal tuning parameter over the class we derive the fastest bounds within the family. Of course, this optimization procedure depends on the regression function f and leads to an oracle in the frequentist framework. In a second step, we put a prior on the tuning parameter and derive the posterior concentration rate of the constructed hierarchical Gaussian prior distribution. The results of the Bayesian inference are furthermore put into relation with the frequentist problem of estimating the regression function f. Considering typical smoothness classes, we show that a full data-driven Bayes estimate derived from this hierarchical prior distribution can attain minimax optimal rates of convergence and is hence adaptive.

Keywords: adaptive estimation, Bayesian inference, Gaussian sequence, minimax theory, nonparametric estimation.
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Bibliography

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