Intervention in Ornstein-Uhlenbeck SDEs

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Abstract

We introduce a notion of intervention for stochastic differential equations and a corresponding causal interpretation. For the case of the Ornstein-Uhlenbeck SDE, we show that the SDE resulting from a simple type of intervention again is an Ornstein-Uhlenbeck SDE. We discuss criteria for the existence of a stationary distribution for the solution to the intervened SDE. We illustrate the effect of interventions by calculating the mean and variance in the stationary distribution of an intervened process in a particularly simple case.

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