# Adaptive confidence sets from Bayes procedures

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**Abstract**

Adaptive techniques for nonparametric estimation have been widely studied in the literature and many rate-adaptive results have been provided for a variety of statistical problems. However an adaptive estimator without any knowledge of its uncertainty is rather uninformative, since one knows that the estimator is optimally close to the true function, but has no information about the actual distance.
In the Bayesian framework credible sets can be constructed to quantify the uncertainty in the posterior distribution. Due to the recent developments of Bayesian computational methods constructing credible sets can be easier than constructing confidence sets from frequentist estimators. The frequentist coverage of credible sets describes to what extent credible sets can be viewed as frequentist confidence sets.
We consider the problem of constructing Bayesian based confidence sets that are adaptive in $L^2$-loss over a continuous scale of Sobolev classes in the Gaussian White noise model. We show that both the hierarchical Bayes and marginal likelihood empirical Bayes approaches lead to credible sets with asymptotic coverage zero for certain oddly behaving functions. Then we give a new empirical Bayes method based on the results of [1], which solves this problem and provides uniform and adaptive confidence sets over a whole collection of Sobolev classes.

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**Bibliography**

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