Directed Random Graphs and Convergence to the Tracy-Widom Distribution

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Abstract

We consider a directed random graph on the 2-dimensional integer lattice, placing independently, with probability p, a directed edge between any pair of distinct vertices (i_1, i_2) and (j_1, j_2), such that $i_1 \leq j_1$ and $i_2 \leq j_2$. Let $L_{n,m}$ denote the maximum length of all paths contained in an $n \times m$ rectangle. The asymptotic distribution for a centered/scaled version of $L_{n,m}$, for fixed m, as $n \to \infty$, was derived in [2]. Here, we address the problem of finding the limit when both n and m tend to infinity, so that $m \sim n^a$. We make a sequence of transformations in order to exhibit a resemblance of our model to a last passage percolation model. This requires the use of suitably defined regenerative points (called skeleton points), together with a number of pathwise and probabilistic bounds. Making use of a Komlós-Major-Tusnády coupling, as in [1], with a last-passage Brownian percolation model, we are able to prove that, for $a < 3/14$, the asymptotic distribution is the Tracy-Widom distribution.

Keywords: Random graph, Last passage percolation, Strong approximation, Tracy-Widom distribution
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