Parameter estimation for discretely observed fractional Ornstein-Uhlenbeck process of the second kind

Lauri Viitasaari[1] and Ehsan Azmoodeh[2]

[1] *Department of Mathematics and System Analysis, Aalto University School of Science, Finland*
[2] *Faculté des Sciences, de la Technologie et de la Communication, Université du Luxembourg, Luxembourg*

Abstract

Let W_t be a standard Brownian motion. Classical Ornstein-Uhlenbeck processes can be obtained via solution of the Langevin equation $\mathrm{d}X_t = -\theta X_t \mathrm{d}t + \mathrm{d}W_t$ or via Lamperti transform $X_t^{(\alpha)} = e^{-\theta t} W_{\alpha e^{2\theta t}}$. With a particular choice $\alpha = \frac{1}{2\theta}$ these two processes are the same in a sense that they have the same finite dimensional distributions. This is not the case however if one replaces Brownian motion W_t with fractional Brownian motion B_t^H. In particular, the process arising from Lamperti transform can be viewed as a solution to Langevin type equation $\mathrm{d}X_t = -\theta X_t \mathrm{d}t + \mathrm{d}Y_t$ with a noise Y given by

$$Y_t = \int_0^t e^{-s} \mathrm{d}B^H_{He^{\frac{t}{H}}}.$$

As a result we obtain two different fractional Ornstein-Uhlenbeck processes depending on the approach. The solution to Langevin equation $\mathrm{d}X_t = -\theta X_t \mathrm{d}t + \mathrm{d}B_t^H$ is referred to fractional Ornstein-Uhlenbeck process of the first kind and the process arising from Lamperti transform is referred to fractional Ornstein-Uhlenbeck process of the second kind.
An interesting problem in mathematical statistics is to estimate the unknown parameter θ. One approach is to consider LSE estimator based on Skorokhod integrals. This is considered by Hu and Nualart [3] for first kind process and by Azmoodeh and Morlanes [1] for second kind process. However, divergence integrals cannot be computed from the path of the process. Another approach is to observe the path of the process and estimate the unknown parameter θ directly from the observations. We consider discretely observed second kind process and find strongly consistent estimator. We also introduce an estimator based on generalised quadratic variations for Hurst parameter H. Moreover, we derive central limit theorems for our estimators. Similar results for first kind process is derived by Brouste and Iacus [2].

Keywords: fractional Ornstein-Uhlenbeck processes, Langevin equation, parameter estimation, central limit theorem
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