Multifractal products of geometric stationary processes

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Abstract

This is joint work with D. Denisov (Cardiff University).

Multifractal and monofractal models have been used in many applications in hydrodynamic turbulence, finance, genomics, computer network traffic, etc. (see, for example, [7]). There are many ways to construct random multifractal models ranging from simple binomial cascades to measures generated by branching processes and the compound Poisson process ([4] - [7]).

Anh, Leonenko and Shieh ([1]-[3]) and Leonenko and Shieh [8] considered multifractal products of stochastic processes as defined in [9], but they provide a new interpretation of the conditions on the characteristics of geometric stationary processes in terms of the moment generating functions.

We investigate the properties of multifractal products of geometric Gaussian processes with possible long-range dependence and geometric Ornstein-Uhlenbeck processes driven by Lévy motion and their finite and infinite superpositions. We present the general conditions for the $\mathcal{L}_q$ convergence of cumulative processes to the limiting processes and investigate their q-th order moments and Rényi functions, which are nonlinear, hence displaying the multifractality of the processes as constructed. We also establish the corresponding scenarios for the limiting processes, such as log-normal, log-gamma, log-tempered stable or log-normal tempered stable scenarios.

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