

Statistics for Stochastic Differential Equations - Two Approaches

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Abstract

For discrete-time observations of the solution to a stochastic differential equation, there is usually no explicit expression for the likelihood function, which is a product of transition densities. Therefore, the likelihood function must be approximated. A brief review will be given of a broad spectrum of approximation methods. Two approaches will be presented in detail. Martingale estimating functions are a simple way of approximating likelihood inference that provides estimators which are easy to calculate. These estimators are generally consistent, and if the estimating function is chosen optimally, they are efficient in a high frequency asymptotic scenario, where the sampling frequency goes to infinity. At low sampling frequencies, efficient estimators can be obtained by more accurate approximations to likelihood inference based on simulation methods, including both the stochastic EM-algorithm and Bayesian approaches like the Gibbs sampler. These methods are much more computer intensive. Simulation of diffusion bridges plays a central role. Therefore this highly non-trivial problem has been investigated actively over the last 10 years. A simple method for diffusion bridge simulation will be presented and applied to likelihood inference for stochastic differential equations.